# Transient heat and mass transfer in film absorption of finite depth with nonhomogeneous boundary conditions

A. HAJJI

Institut Agronomique et Veterinaire Hassan II, Department de Genie Industriel Alimentaire, B.P. 6202 Rabat-Instituts, Morocco

and

# W. M. WOREK†

University of Illinois at Chicago, Department of Mechanical Engineering (m/c 251), P.O. Box 4348, Chicago, IL 60680, U.S.A.

(Received 2 July 1991 and in final form 11 September 1991)

Abstract—A theoretical analysis of transient combined heat and mass transfer processes that occur in liquid desiccant film absorption is presented. Analytical expressions are derived for the temperature and concentration in a stagnant film of finite depth. Comparison with experimental data allows the determination of the effective Lewis number. Using the analytical expressions, the transient variation of the heat and mass rates and the transient dependence of the dimensionless average temperature with Lewis number and the dimensionless enthalpy of absorption are presented.

# **1. INTRODUCTION**

GAS AND vapor absorption occurs in many chemical and energy systems. In particular, absorption heating and cooling systems have received a lot of interest because they can be powered by solar energy or by a low temperature waste heat source. Numerous studies have been done simulating and optimizing absorption systems. However, much less attention has been given to understanding the absorption mechanisms despite the important effect of the heat and mass transfer rates on the performance of absorption systems.

Grossman [1] reviewed the state of knowledge on film absorption. Additional studies on absorption heat and mass transfer are given in refs. [2-8]. Le Goff et al. [3] presented the results of a numerical study of an absorbing film falling along an adiabatic wall. The same authors [4] developed an approximate solution and compared it to their more rigorous numerical results [3]. Kashiwagi et al. [5] used a holographic interferometer to experimentally study the unsteady heat and mass transfer processes in a stagnant layer of an aqueous lithium bromide (LiBr) solution absorbing water vapor. In their experiment they had a stagnant film of liquid initially at a temperature  $T_0$  and concentration  $C_0$  which was suddenly put in contact with the gas phase absorbate which was at a constant pressure different from the equilibrium pressure at  $(T_0, C_0)$ . By comparing the numerical results, obtained by solving the combined heat and

mass transfer equations, with experimental profiles of temperature and concentration, they determined an effective Lewis number. Experiments conducted by Zawacki *et al.* [6] investigated the absorption of aqueous LiBr solutions. They showed that the thickness of the liquid desiccant film has an important effect on the absorption rate.

In this paper, a theoretical analysis of the transient, coupled heat and mass transfer processes for the basic situation described above is presented. Analytical expressions for the temperature and concentration distributions are then obtained using a Fourier series expansion technique. The film depth is taken into account and nonhomogeneous boundary conditions at the lower surface of the film are considered. In addition to its usefulness in a hydrodynamic analysis, the solution can be adapted to the problem of steady absorption by a falling film over a heated plate, such as a solar collector plate. Comparison with experimental data allows an estimation of the effective Lewis number and the calculation of the Nusselt and Sherwood numbers.

#### 2. EFFECT OF THE FILM THICKNESS

# 2.1. Governing equations

We consider the system shown in Fig. 1 neglecting heat transfer in the gas phase and the heat of dilution. Absorption equilibrium is assumed at the gas -liquid interface. The boundary conditions at the lower surface of the film can be either a specified temperature  $T_s$  or a specified heat flux  $q_s$  with specified mass flux

<sup>†</sup>To whom correspondence should be addressed.

NOMENCLATURE									
G	$v_n$ coefficient introduced in equation (5a)	Greek symbols							
t	coefficient introduced in equation (5b)	α	thermal diffusivity						
(	concentration	ζ	dimensionless position coordinate						
(	T <sub>p</sub> liquid desiccant specific heat	$\theta$	dimensionless temperature						
L	D mass diffusivity	λ	dimensionless enthalpy of absorption						
a	liquid desiccant film thickness	$\mu$	eigenvalue						
f	equilibrium function	$\rho$	liquid desiccant density						
Δ	<i>H</i> specific enthalpy of absorption	τ	dimensionless time						
h	enthalpy	$\phi$	eigenfunction for dimensionless						
I	coefficients introduced in equations (18a)		temperature						
	and (22a)	ψ	eigenfunction for dimensionless						
Ι	<sup>2</sup> coefficients introduced in equations (18a)		concentration						
	and (22a)	ω	dimensionless concentration.						
k	thermal conductivity of the desiccant film								
	and index for determining the								
	cigenvalues	Subscrip	pts						
L	Le Lewis number	c	equilibrium						
Ν	<i>A</i> nondimensional mass transfer flux	f	fluid or saturated liquid						
p	pressure	g	gas phase						
Q	2 nondimensional heat transfer flux	i	initial values						
4	heat transfer rate	п	index for the eigenvalues and						
4	differential heat of solution		eigenfunction						
r	coefficient defined by equation (13)	0	initial quantity						
7	f temperature	р	particular solution						
t	time	S	surface						
2	position.	$H_2O$	water phase.						

 $m_{\rm s}$ . Therefore the governing equations and boundary conditions are:

Energy equation

$$\partial T/\partial t = \alpha \, \partial^2 T/\partial z^2 \tag{1a}$$

Mass equation

$$\partial C/\partial t = D \,\partial^2 C/\partial z^2 \tag{1b}$$

Initial conditions

$$T(z,0) = T_0, \quad C(z,0) = C_0$$
 (2a,b)

Boundary conditions

at z = 0:

$$p = f_{eq}(T_i, C_i), \quad k \frac{\partial T}{\partial z} \bigg|_{z=0} = \rho D \frac{\partial C}{\partial z} \bigg|_{z=0} (\Delta H);$$
(3)

at z = d:

$$T = T_{s} \text{ or } -k \frac{\partial T}{\partial z}\Big|_{z=d} = q_{s}$$
  
and 
$$-D \frac{\partial C}{\partial z}\Big|_{z=d} = m_{s}$$
(4)

where  $f_{\rm eq}$  is the equilibrium function and  $\Delta H$  is the

# Vapor Phase (air-absorbate)



FIG. 1. Mathematical model for absorption by a film of finite depth.

specific enthalpy of absorption. A more convenient form of the above equations is obtained by assuming that the equilibrium function  $f_{cq}$  is linearly dependent on the concentration and introducing the following dimensionless variables :

$$\zeta = z/d, \quad \tau = \alpha t/d^2, \quad \theta = \frac{(T - T_0)}{(T_e - T_0)}, \quad \omega = \frac{(C - C_0)}{(C_e - C_0)}$$

where  $T_e$  and  $C_e$  satisfy the equilibrium relations

$$p = f_{eq}(T_e, C_0) = f_{eq}(T_0, C_e).$$
 (5)

The dimensionless form of equations (1)-(4) become

$$\partial \theta / \partial \tau = \partial^2 \theta / \partial \zeta^2 \tag{1a*}$$

$$\partial \omega / \partial \tau = Le \, \partial^2 \omega / \partial \zeta^2 \tag{1b*}$$

at  $\tau = 0$ ,  $\theta(\zeta, 0) = 0$ ,  $\omega(\zeta, 0) = 0$  (2a\*,b\*)

at 
$$\zeta = 0$$
,  $\theta_{i} + \omega_{i} = 1$ ,  $\left. \frac{\partial \theta}{\partial \zeta} \right|_{\zeta = 0} = Le \left. \lambda \frac{\partial \omega}{\partial \zeta} \right|_{\zeta = 0}$  (3\*)  
at  $\zeta = 1$ ,  $\theta(1, \tau) = \theta_{s}$   
or  $\left. \frac{\partial \theta}{\partial \zeta} \right|_{\zeta = 1} = Q$ ,  $\left. \frac{\partial \omega}{\partial \zeta} \right|_{\zeta = 1} = M$ , (4\*)

where Le is the Lewis number,  $\lambda$  is the dimensionless enthalpy of absorption, and Q and M are the dimensionless heat and mass transfer fluxes defined by:

$$Le = D/\alpha; \quad \lambda = \frac{\Delta H(C_c - C_0)}{C_p(T_c - T_0)};$$
$$Q = -\frac{q_s d}{k(T_c - T_0)} \quad \text{and} \quad M = -\frac{m_s d}{D(C_c - C_0)}.$$

The dimensionless equations show that the analytical solution depends only on the boundary conditions  $\theta_s$  or Q and M, the Lewis number and the dimensionless enthalpy of absorption.

#### 2.2. Method of solution

The linear character of the governing equations suggests that analytical methods, such as Fourier scries expansion (separation of variables) or Laplace transform techniques, could be used.

To apply the Fourier series expansion method, we introduce the expressions :

$$\theta = \theta_{\rm p}(\zeta, \tau) + \sum_{n=0}^{\infty} a_n \phi_n(\zeta) \exp\left(-\mu_n^2 \tau\right)$$
 (5a)

$$\omega = \omega_{p}(\zeta, \tau) - \sum_{n=0}^{\infty} b_{n} \psi_{n}(\zeta) \exp\left(-\mu_{n}^{2} \tau\right), \quad (5b)$$

where  $\theta_p$  and  $\omega_p$  are particular solutions which satisfy the nonhomogeneous boundary conditions. The eigenfunctions  $\phi_n$  and  $\psi_n$  satisfy the differential equations

$$\phi_n'' = -\mu_n^2 \phi_n \tag{6a}$$

$$\psi_n'' = -\mu_n^2 \psi_n / Le, \qquad (6b)$$

and the homogeneous boundary conditions,

at 
$$\zeta = 0$$
,  $a_n \phi_n(0) - b_n \psi_n(0) = 0$ ,  
 $a_n \phi'_n(0) + Le \, \lambda b_n \psi'_n(0) = 0$ ; (7)

at  $\zeta = 1$ ,  $\phi_n(1) = 0$  or  $\phi'_n(1) = 0$ ,  $\psi'_n(1) = 0$ . (8)

The eigenvalues  $\mu_n$  are obtained by solving the characteristic equation derived by equating the discriminant of the set of equations, given by equation (7), to zero. That is

Le 
$$\lambda \phi_n(0) \psi'_n(0) + \phi'_n(0) \psi_n(0) = 0.$$
 (9)

The orthogonality condition, derived using a procedure similar to that in ref. [2], gives the following condition:

$$a_n a_m \int_0^1 \phi_n \phi_m \, \mathrm{d}\zeta + \lambda b_n b_m \int_0^1 \psi_n \psi_m \, \mathrm{d}\zeta = 0 \quad n \neq m.$$
(10)

The initial conditions given by equations (2a\*,b\*) become

$$\theta_{p}(\zeta,0) = -\sum_{n=0}^{\infty} a_{n}\phi_{n}(\zeta)$$
(11a)

$$\omega_{\mathfrak{p}}(\zeta,0) = \sum_{n=0}^{\infty} b_n \psi_n(\zeta). \tag{11b}$$

Using the orthogonality condition given by equation (10) yields

$$a_{n} = \frac{\left\{-\int_{0}^{1} \phi_{n} \theta_{p}(\zeta, 0) \, \mathrm{d}\zeta + \lambda r \int_{0}^{1} \psi_{n} \omega_{p}(\zeta, 0) \, \mathrm{d}\zeta\right\}}{\left\{\int_{0}^{1} \phi_{n}^{2} \, \mathrm{d}\zeta + \lambda r^{2} \int_{0}^{1} \psi_{n}^{2} \, \mathrm{d}\zeta\right\}}$$
(12a)

$$b_n = ra_n, \tag{12b}$$

where

$$r = \phi_n(0)/\psi_n(0).$$
 (13)

Solving equation (6b) using the boundary condition given by equation (8) yields

$$\psi_n(\zeta) = \cos\left[\frac{\mu_n}{\sqrt{Le}}(1-\zeta)\right].$$
 (14)

Expressions for the characteristic equation and the eigenfunctions are derived in the two cases of specified temperature and specified heat flux.

2.2.1. Specified wall temperature. Solving equation (6a) using the constant temperature boundary condition, given by equation (8), yields

$$\phi_n(\zeta) = \sin\left[\mu_n(1-\zeta)\right]. \tag{15}$$

Therefore equation (9) becomes

$$(1 + \lambda \sqrt{Le}) \cos \{\mu_n [(1/\sqrt{Le}) + 1]\} + (1 - \lambda \sqrt{Le}) \cos \{\mu_n [(1/\sqrt{Le}) - 1]\} = 0.$$
(16)

Solutions to this characteristic equation can be easily

found in some particular cases. For k = 0, 1, 2, ...and for  $\mu_n$  positive, we have

$$Le = 1, \qquad \mu_n = \pm 1/2 \cos^{-1} \left[ \frac{\lambda - 1}{\lambda + 1} \right] + 2k\pi$$
$$\lambda \sqrt{Le} = 1, \qquad \mu_n = \left[ \frac{\pm \pi/2 + 2k\pi}{1 + (1/\sqrt{Le})} \right]$$
$$\lambda \sqrt{Le} \gg 1, \qquad \mu_n = \{k\pi, k\pi \sqrt{Le}\}$$
$$\lambda \sqrt{Le} \ll 1, \qquad \mu_n = \left\{ \frac{(2k+1)\pi}{2}, \frac{(2k+1)\pi \sqrt{Le}}{2} \right\}.$$

The particular solutions  $\theta_p$  and  $\omega_p$  are given by

$$\theta_{\rm p} = \theta_{\rm s} + \lambda \cdot Le \cdot M(\zeta - 1) \tag{17a}$$

$$\omega_{\rm p} = 1 - \theta_{\rm s} + \lambda \cdot Le \cdot M + M\zeta \tag{17b}$$

and the coefficients  $a_n$  and  $b_n$  are given by

$$a_n = 2 \left\{ \frac{-I_1 + \lambda r I_2}{\left(1 - \frac{\sin\left(2\mu_n\right)}{2\mu_n}\right) + \lambda r^2 \left(1 + \frac{\sin\left(2\mu_n/\sqrt{Le}\right)}{(2\mu_n/\sqrt{Le})}\right)} \right\},$$
(18a)

where

$$I_{1} = \frac{(1 - \cos \mu_{n})}{\mu_{n}} \theta_{s} + \frac{(\mu_{s} \cos \mu_{n} - \sin \mu_{n})}{\mu_{n}^{2}} \lambda Le \cdot M,$$

$$I_{2} = \frac{\sin (\mu_{n}/\sqrt{Le})}{\mu_{n}/\sqrt{Le}} (1 - \theta_{s}) + \left[\frac{1 - \cos (\mu_{n}/\sqrt{Le})}{\mu_{n}^{2}/Le} + \lambda Le \frac{\sin (\mu_{n}/\sqrt{Le})}{\mu_{n}/\sqrt{Le}}\right] M$$

and

$$b_n = \left\{ \frac{\sin \mu_n}{\cos \left( \mu_n / \sqrt{Le} \right)} \right\} a_n.$$
(18b)

2.2.2. Specified wall heat flux. Solving equation (6a) using the specified flux boundary condition, given by equation (8), yields

$$\phi_n(\zeta) = \cos\left[\mu_n(1-\zeta)\right]. \tag{19}$$

Therefore equation (9) becomes

$$(1 + \lambda \sqrt{Le}) \sin \{\mu_n [1/\sqrt{Le} + 1]\}$$
  
-  $(1 - \lambda \sqrt{Le}) \sin \{\mu_n [1/\sqrt{Le} - 1]\} = 0.$  (20)

Solutions to this characteristic equation can be easily found in some particular cases. For k = 0, 1, 2, ... and for  $\mu_n$  positive, we have

$$Le = 1, \qquad k\pi/2$$

$$\lambda\sqrt{Le} = 1, \qquad \mu_n = \frac{k\pi}{[1 + (1/\sqrt{Le})]}$$

$$\lambda\sqrt{Le} \gg 1, \qquad \mu_n = \{(k+1/2)\pi, k\pi\sqrt{Le}\}$$

$$\lambda\sqrt{Le} \ll 1, \qquad \mu_n = \left\{k\pi, \frac{(2k\pm1)\pi}{2}\sqrt{Le}\right\}.$$

The particular solutions for nondimensional temperature and concentration,  $\theta_p$  and  $\omega_p$ , are given by

$$\theta_{p}(\zeta,\tau) = \frac{(Q-\lambda Le M)}{1+\lambda} \{\tau + \zeta[(\zeta/2) - 1]\} + Q\zeta \quad (21a)$$
$$\omega_{p}(\zeta,\tau) = 1 - \frac{(Q-\lambda Le M)}{1+\lambda} \{\tau + (\zeta/Le)[(\zeta/2) - 1]\} + M\zeta$$

(21b)

and the coefficients  $a_n$  and  $b_n$  are given by

$$a_n = 2 \left\{ \frac{(-I_1 + \lambda r I_2)}{\left(1 + \frac{\sin\left(2\mu_n\right)}{2\mu_n}\right) + \lambda r^2 \left(1 + \frac{\sin\left(2\mu_n/\sqrt{Le}\right)}{(2\mu_n/\sqrt{Le})}\right)} \right\},$$
(22a)

where

$$I_{1} = \frac{Q - \lambda Le M}{1 + \lambda} \left( \frac{\mu_{n} \cos \mu_{n} - \sin \mu_{n}}{\mu_{n}^{3}} \right) + \frac{1 - \cos \mu_{n}}{\mu_{n}^{2}} Q,$$

$$I_{2} = \frac{\sin \left( \frac{\mu_{n}}{\sqrt{Le}} \right)}{\frac{\mu_{n}}{\sqrt{Le}}}$$

$$- \frac{Q - \lambda Le M}{(1 + \lambda)Le} \left( \frac{\frac{\mu_{n}}{\sqrt{Le}} \cos \left( \frac{\mu_{n}}{\sqrt{Le}} \right) - \sin \left( \frac{\mu_{n}}{\sqrt{Le}} \right)}{(\frac{\mu_{n}}{\sqrt{Le}})^{3}} \right)$$

$$+ \frac{1 - \cos \left( \frac{\mu_{n}}{\sqrt{Le}} \right) M}{\mu_{n}^{2}/Le}$$

and

$$b_n = \frac{\cos \mu_n}{\cos \left(\mu_n / \sqrt{Le}\right)} a_n.$$
(22b)

The first eigenvalue is 0 and the corresponding coefficients are

$$a_0 = \frac{(Q - \lambda Le M)(Le + \lambda)}{3 Le \lambda(\lambda + 1)} - \frac{Q - \lambda M}{2\lambda} \quad (23a)$$

$$b_0 = a_0. \tag{23b}$$

#### 3. COMPARISON WITH EXPERIMENTAL DATA

Kashiwagi *et al.* [5] studied the absorption of water vapor by aqueous solutions of LiBr using combined holography and thermometry techniques. Their experimental set-up is similar to the basic system shown in Fig. 1. Their data were obtained for the following conditions: ambient temperature,  $T_0$ , of  $20^{\circ}$ C; initial concentration,  $C_0$ , of 50%; vapor pressure, p, of 2.38 kPa; and thermal diffusivity  $\alpha = 1.40 \times 10^{-7}$  m<sup>2</sup> s<sup>-1</sup>. The equilibrium relation for LiBr-water absorption is derived using the graphical data of ref. [7] and is written in the form

$$T(^{\circ}C) = -161.1 \cdot C + 124.16 \quad (p = 2.38 \text{ kPa}).$$
 (24)

2104

0.5

Additional physical properties of LiBr solutions are given in ref. [8]. In particular, the heat of absorption can be calculated from the differential heat of solution  $q'_d$  defined in ref. [8] by

$$q'_{\rm d} \equiv h_{\rm H,O} - h_{\rm f,H,O},$$
 (25)

where  $h_{\rm H,O}$  is the enthalpy of water in the LiBr solution and  $h_{\rm f,H_2O}$  is the enthalpy of pure liquid water at the same temperature. For the conditions mentioned above we take

$$\Delta H \equiv h_{\rm g,H_2O} - h_{\rm H_2O} = 2306.9 \text{ kJ kg}^{-1} \qquad (26)$$

which leads to  $\hat{\lambda} = 6.8$ .

The profile of the measured dimensionless temperature when plotted vs  $\zeta/(2\sqrt{\zeta})$  is well represented by an error function expression as predicted by the theory. Figure 2 shows that for Le = 0.015, the present solution given by equation (5a) agrees well with the experimental data of Kashiwagi et al., who also predicted the same value of Le using a numerical approach. The small deviation from the theory can be attributed to the dependence of Le on concentration as confirmed by the use of solutions with different concentrations. Also, the time variation of the interface temperature, especially at small values of time, which contradicts the theory, may be due to the fact that the assumption of equilibrium is not valid at the beginning of absorption, or to a probable decrease of the vapor pressure below the gas phase pressure near the interface when the absorption process begins.

Table 1 gives the first 20 eigenvalues for Le = 0.015and  $\lambda = 6.8$ . The coefficients  $a_n$  and  $b_n$  are also presented for the case of homogeneous boundary conditions. Typical profiles of the temperature and con-

Le≕0.020 Dimensionless Temperature,  $\theta$ 0 0.015  $\lambda = 6.8$ Le = 0.0100.3 Le=0.005 Le=0.001 0.2 0.1 0.0 1.0 2.0 0.0 0.5 1.5 2.5 3.0 5/257

FIG. 2. Determination of the effective Lewis number by comparison with experimental results of ref. [5].

centration are shown in Figs. 3(a) and (b). For small times, the effect of the lower boundary is not felt and the profiles approach the analytical solution for a semi-infinite medium. In the case of an isothermal wall, the average temperature reaches a maximum value before decreasing to zero. In the case of an insulated wall,  $\theta_{av}$  increases steadily towards the asymptotic value  $\lambda/(\lambda + 1)$ . The variations of the heat and mass fluxes with time at the interface are shown in Figs. 4(a) and (b). We notice very sharp gradients of temperature and concentration at the beginning of absorption which decreases very rapidly to zero. The effect of Lewis number and the effect of the heat of absorption are illustrated in Figs. 5(a), (b) and 6(a)

Table 1. First 20 eigenvalues and the corresponding coefficients  $a_n$  and  $b_n$  for Le = 0.015 and  $\lambda = 6.8$ 

	Prescribed temperature			Prescribed flux		
n	$\mu_n$	a <sub>n</sub>	b <sub>n</sub>	$\mu_n$	a <sub>n</sub>	b <sub>n</sub>
0	0.17453	1.05341	1.25901	0.00000	0.87179	0.87179
1	0.52239	0.32995	-0.38076	0.33612	-0.24149	0.24723
2	0.86690	0.16484	0.17944	0.67568	-0.20627	-0.22338
3	1.20684	0.07211	-0.07410	1.01984	-0.16741	0.19239
4	1.54313	0.00460	0.00460	1.36750	-0.13410	-0.16001
5	1.87909	-0.03999	0.04079	1.71654	-0.10733	0.12846
6	2.21829	-0.06115	-0.06588	2.06459	-0.08421	-0.09757
7	2.56209	-0.06596	0.07547	2.40944	-0.06069	0.06641
8	2.90954	-0.06284	-0.07484	2.74976	-0.03372	-0.03479
9	3.25856	-0.05664	0.06786	3.08625	-0.00459	0.00459
10	3.60677	-0.04859	-0.05652	3.42210	0.02015	0.02049
11	3.95196	-0.03780	0.04158	3.76094	0.03503	-0.03755
12	4.29266	-0.02288	-0.02370	4.10437	0.04071	0.04638
13	4.62937	-0.00458	0.00459	4.45158	0.04094	-0.04865
14	4.96512	0.01261	0.01278	4.80056	0.03849	0.04617
15	5.30362	0.02407	-0.02567	5.14894	0.03428	-0.04002
16	5.64667	0.02924	0.03315	5.49444	0.02773	0.03066
17	5.99364	0.03029	-0.03591	5.83553	0.01774	-0.01846
18	6.34256	0.02916	0.03500	6.17249	0.00457	0.00458
19	6.69108	0.02656	-0.03111	6.50816	-0.00862	0.00872



FIG. 3. (a) Transient profiles of dimensionless temperature and concentration for the case of an insulated wall. (b) Transient profiles of dimensionless temperature and concentration for the case of an isothermal wall  $(\theta_s = 0)$ .



FIG. 4. (a) Transient profiles of dimensionless heat and mass flux at the interface ( $\zeta = 0$ ) for the case of an insulated wall. (b) Transient profiles of dimensionless heat and mass flux at the interface ( $\zeta = 0$ ) for the case of an isothermal wall ( $\theta_s = 0$ ).



FIG. 5. (a) Effect of Lewis number on the transient dimensionless average temperature for the case of an insulated wall for  $\lambda = 6.8$ . (b) Effect of Lewis number on the transient dimensionless average temperature for the case of an isothermal wall ( $\theta_s = 0$ ) for  $\lambda = 6.8$ .

2106



FIG. 6. (a) Effect of dimensionless differential heat of absorption on the transient dimensionless average temperature for the case of an insulated wall for Le = 0.01. (b) Effect of dimensionless differential heat of absorption on the transient dimensionless average temperature for the case of an isothermal wall ( $\theta_s = 0$ ) for Le = 0.01.

and (b), respectively. The interface temperature changes according to the expressions  $\lambda \sqrt{Le}$  $(1 + \lambda \sqrt{Le})$ . The maximum average temperature increases with Le and with  $\lambda$ . The effect of the heat of absorption is more pronounced in the case of an insulated wall due to the heat removed at the lower boundary than in the case of an isothermal wall. Figures 7(a) and (b) show the influence of a nonhomogeneous boundary condition. The prescribed flux at the lower boundary causes a decrease in the average temperature at small times, due to desorption of the solution, before the external heat input dominates and a linear increase in temperature according to the steady state solution, given by equation (21), is observed. The profiles of heat and mass flux at the interface are not significantly affected by Q. At small times, the temperature and concentration gradients

remain large and are almost independent of Q before decreasing to an asymptotic value determined by the steady-state solutions given by equations (21a) and (21b).

## 4. CONCLUSIONS

Transient heat and mass transfer processes that occur in film absorption are analyzed. The film thickness is taken into account and nonhomogeneous boundary conditions are used. Analytical expressions for the temperature and the concentration are determined. Comparison with experimental data gives a value of 0.015 for the effective Lewis number in the case of absorption of water vapor by a lithium bromide solution.



FIG. 7. (a) Effect of the imposed heat flux on the transient dimensionless average temperature (M = 0). (b) Effect of the imposed heat flux on the transient dimensionless heat flux at the interface (M = 0).

Acknowledgement—The lead author would like to acknowledge the International Foundation of Science (IFS), located in Stockholm, Sweden, for its financial support.

#### REFERENCES

- G. Grossman, Heat and mass transfer in film absorption. In Handbook of Heat and Mass Transfer, Chapter 6. Gulf, Houston, TX (1986).
- 2. G. Grossman, Simultaneous heat and mass transfer in film absorption under laminar flow, *Int. J. Heat Mass Transfer* **25**, 357–371 (1983).
- H. Le Goff, A. Ramdane et P. Le Goff, Modelisation des transfers couples de chaleur et de masse dans l'absorption gaz-liquide en film ruisselant laminaire, *Int. J. Heat Mass Transfer* 28, 2005-2017 (1985).
- 4. H. Le Goff, A. Ramdane et P. Le Goff, Un model simple de la penetration couplee de chaleur et de masse dans

l'absorption gas-liquide en film ruisselant laminaire, Int. J. Heat Mass Transfer 29, 625-634 (1986).

- T. Kashiwagi, Y. Kurosaki and I. Naikai, Heat and mass diffusion in the absorption of water vapor by aqueous solutions of lithium bromide, *Trans. Jap. Assoc. Refrigeration* 1, 89–98 (1984).
- T. Zawacki, R. A. Macriss and W. F. Rush, The effect of additives on the level of instability of gas/liquid interfaces : the absorption of water vapor concentrated lithium bromide solutions in falling film and open-channel absorbers. Presented at the 75th Meeting of the A.I.Ch.E., Detroit, MI, 30 June (1973).
- 7. ASHRAE Handbook of Fundamentals, p. 17.70. American Society of Heating Refrigeration and Air-Conditioning Engineers, Atlanta, GA (1985).
- 8. H. Lower, Thermodynamiche und Physikalische Eigen-Schaften der wassrigen Lithium-bromid-Losung, Ph.D. Thesis, Karlsruhe, West Germany (1960).

## TRANSFERT VARIABLE DE CHALEUR ET DE MASSE DANS L'ABSORPTION EN FILM D'EPAISSEUR FINIE AVEC DES CONDITIONS AUX LIMITES NON HOMOGENES

**Résumé**—On présente une analyse théorique des mécanismes combinés de transfert variable de chaleur et de masse qui opèrent dans un film liquide dessiccatif. Des expressions analytiques sont obtenues pour la température et la concentration dans un film stagnant d'épaisseur finie. Une comparaison avec les données expérimentales permet la détermination du nombre de Lewis effectif. On présente à l'aide d'expressions analytiques, la variation dans le temps des flux de chaleur et de masse et la dépendance variable de la température adimensionnelle moyenne au nombre de Lewis et à l'enthalpie adimensionnelle d'absorption.

#### TRANSIENTER WÄRME- UND STOFFTRANSPORT BEI DER ABSORPTION IN EINEM FILM ENDLICHER DICKE MIT NICHTHOMOGENEN RANDBEDINGUNGEN

Zusammenfassung—Es wird eine theoretische Analyse des transienten gekoppelten Wärme- und Stofftransports, wie er bei der Filmabsorption in einem flüssigen Trocknungsmittel auftritt, vorgestellt. Für die Temperatur und Konzentration in einem ruhenden Film endlicher Tiefe werden analytische Ausdrücke abgeleitet. Der Vergleich mit experimentellen Daten erlaubt die Bestimmung der effektiven Lewis-Zahl. Unter Verwendung der analytischen Ausdrücke wird die transiente Veränderung der Wärme- und Stoffströme, die transiente Abhängigkeit der dimensionslosen Mitteltemperatur von der Lewis-Zahl sowie der dimensionslosen Enthalpie der Absorption vorgestellt.

# НЕСТАЦИОНАРНЫЙ ТЕПЛО- И МАССОПЕРЕНОС ПРИ ПОГЛОЩЕНИИ ПЛЕНКОЙ КОНЕЧНОЙ ТОЛЩИНЫ В СЛУЧАЕ НЕОДНОРОДНЫХ ГРАНИЧНЫХ УСЛОВИЙ

Аннотация — Анализируется нестационарные взаимосвязанные процессы тепло- и массопереноса при поглощении жидкой пленкой осушителя. Получены аналитические выражения для температуры и концентрации в неподвижной пленке конечной толщины. Сравнение с экспериментальными данными позволяет определить эффективное число Льюиса. В нестационарном случае приводятся аналитические выражения для изменения скорости тепло- и массопереноса, зависимости безразмерной средней температуры от числа Льюиса, а также для безразмерной энтальпии поглощения.